*Korean J. of Chem. Eng.*, **12**(4), 472-477 (1995)

# A STUDY ON THE ADAPTIVE PREDICTIVE CONTROL METHOD FOR MULTIVARIABLE BILINEAR PROCESSES

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*(Received 4 April 1995 • accepted 21 June 1995)*

**Abstract**—A predictive control method for multivariable bilinear processes is derived based on ARMA model. To identify bilinear process models, we use simple equation error method extended to multivariable system. We can obtain the adaptive predictive controller for multivariable bilinear processes by incorporation of the identification algorithm. Offset compensator is provided to correct for the effects of unmeasured disturbances and model inaccuracies. A filter with a singled parameter is used to correct for the effects of an incorrect model. Results of simulation on multivariable bilinear processes show that the proposed control method has satisfactory performance.

*Key words: Bilinear Models, ARMA Model, Adaptive Control, Predictive Control*

## INTRODUCTION

Much research in the field of process control has been primarily focused on the design of a control system capable of maintaining the process at its optimal steady-state despite changing various operating conditions. Most typical industrial processes are time varing and nonlinear in nature, and in many instances the task of modelling such processes is a very difficult one. This is why many morden control methods that require an exact knowledge of the process cannot be applied satisfactorily to the control of such processes. Moreover such fixed gain control strategies cannot satisfactorily accommodate changes in the operating plant. Thus, it is essential to develop a new control technique applicable to nonlinear system.

Recently, rapid development of digital computer technology has made it possible to implement more sophisticated control methods. The predictive control method was subsequently developed [Clarke et al., 1987a, b; Demircioglu and Clarke, 1993; Kouvaritakis and Rossiter, 1993a, b; Yeo, 1986] and applied successfully to several industrial processes involving multivariable process dynamics [Clarke, 1988]. But, in many practical situations, the operating conditions vary with time, and it is very difficult to obtain any information about the parameters of the process to be controlled. Thus, adaptive predictive control method is believed to be the promising strategy applicable in these situations and has recently received much attention as one of the computer control techniques which meet today's need for more effective control strategy.

Many efforts have been devoted to the extension of existing adaptive control method to predictive control method. Lee and Lee [1983] described the adaptive control scheme for disturbance-free systems using a long term predictor. Martin-Sanchez et al. [1984] proposed a stable adaptive predictive control system. They used an equation error identification method and proved several stability properties. Martin-Sanchez and Shah [1984] have applied the above adaptive predictive control scheme to the control of a pilot scale binary distillation column. Cluett et al. [1985] also used the above adaptive predictive controller in the control of a PVC batch reactor. But, since most of the adapive predictive control methods developed so far are based on the linear system models, they cannot handle nonlinear situations which arise especially in the control of chemical processes. Linear systems are described by linear differential or algebraic equations, and the principle of superposition applies. Nonlinear systems are described by complex nonlinear differential equations and linear approximation methods have been used in the control of the nonlinear system. However, the intrinsic limits of the use of linear models appear more and more evident.

Recently, the class of bilinear models has been introduced as a useful tool for examining many nonlinear phenomena. Yeo [1986] proposed the adaptive predictive control method for single-input single-output bilinear systems using the autoregressive moving average (ARMA) model. Many successful application results summarized by Mohler and Kalodzies [1980] illustrate the effectiveness of the use of bilinear models as approximations of nonlinear systems. The primary objective of the present study is to provide the adaptive predictive control method using multivariable bilinear model which is applicable to more general situations and can be easily implemented on real processes.

## CONTROLLER DESIGN

The multivariable plant to be controlled is assumed to be described by a discrete, bilinear model of the form

$$\mathbf{Y^*(k)} = \sum_{i=1}^{N} [\mathbf{A}_i\mathbf{^*Y(k-i)} + \sum_{j=1}^{m} \mathbf{B}_{ij}\mathbf{^*Y(k-i)}u_j(k-i-T) + \mathbf{C}_i\mathbf{^*U(k-i-T)}] \quad (1)$$

T is the known time delay, but we do not need the exact knowledge of the plant structure. We will simplify the problem by considering one-step ahead prediction.

### 1. Prediction of Output

The prediction of the future outputs $\mathbf{Y^*(k+1)},\cdots,\mathbf{Y^*(k+T)}$ does not require future inputs. Since the present modelling error vector $\mathbf{E(k)}$ given by (2) is known, these predicted future values can be obtained by successive substitutions.

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$$\mathbf{E}(\mathbf{k})=\mathbf{Y}(\mathbf{k})-\mathbf{Y}^*(\mathbf{k}) \tag{2}$$

Now we define

$$\underline{\mathbf{Y}}_0=\begin{bmatrix}\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-1)\\ \vdots \\ \mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N})\end{bmatrix}\in R^{Nnx1}, \qquad \underline{\mathbf{U}}_0=\begin{bmatrix}\mathbf{U}^*(\mathbf{k}-1)\\ \vdots \\ \mathbf{U}^*(\mathbf{k}-\mathbf{N})\end{bmatrix}\in R^{Nnx1}$$

$$\underline{\mathbf{X}}_0=\begin{bmatrix}\mathbf{X}_1\\ \vdots \\ \mathbf{X}_N\end{bmatrix}\in R^{Nmnx1}, \quad \mathbf{X}_i=\begin{bmatrix}\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-1)u_1(k-i)\\ \vdots \\ \mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N})u_m(k-i)\end{bmatrix}\in R^{mnx1}$$

$$\underline{\mathbf{E}}=\begin{bmatrix}\mathbf{A}_1^* & \mathbf{A}_2^* & \cdots & \mathbf{A}_N^*\\ \mathbf{A}_2^* & \mathbf{A}_3^* & \cdots \ \mathbf{A}_N^* & 0\\ \vdots & & \iddots & \vdots\\ \mathbf{A}_N^* & 0 & \cdots & 0\end{bmatrix}\in R^{nNxnN},$$

$$\underline{\mathbf{B}}=\begin{bmatrix}\mathbf{B}_1^* & \mathbf{B}_2^* & \cdots & \mathbf{B}_N^*\\ \mathbf{B}_2^* & \mathbf{B}_3^* & \cdots \ \mathbf{B}_N^* & 0\\ \vdots & & \iddots & \vdots\\ \mathbf{B}_N^* & 0 & \cdots & 0\end{bmatrix}\in R^{nNxnmN},$$

$$\underline{\mathbf{F}}=\begin{bmatrix}\mathbf{C}_1^* & \mathbf{C}_2^* & \cdots & \mathbf{C}_N^*\\ \mathbf{C}_2^* & \mathbf{C}_3^* & \cdots \ \mathbf{C}_N^* & 0\\ \vdots & & \iddots & \vdots\\ \mathbf{C}_N^* & 0 & \cdots & 0\end{bmatrix}\in R^{nNxmN}$$

$\mathbf{B}_i^*=[\mathbf{B}_{i1}^*\ \mathbf{B}_{i2}^* \cdots \mathbf{B}_{im}^*]\in R^{nxmn}$, $\mathbf{G}_i=[0\ \cdots\ 0I0\ \cdots\ 0]\in R^{nxnN}$

$\mathbf{F}_0=\underline{\mathbf{E}}\underline{\mathbf{Y}}_0+\underline{\mathbf{B}}\underline{\mathbf{X}}_0+\underline{\mathbf{F}}\underline{\mathbf{U}}_0\in R^{nNx1}$

Then $\mathbf{Y}^*(\mathbf{k}+\mathbf{T})$ can be written as

$$\mathbf{Y}^*(\mathbf{k}+\mathbf{T})=\mathbf{G}_1\mathbf{F}_0+\mathbf{E}(\mathbf{k})=\mathbf{Q}_0 \tag{3}$$

Using this equation, we have

$$\begin{aligned}\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1)&=\mathbf{A}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})+\cdots+\mathbf{A}_N^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N}+1)+\cdots\\&\quad+\mathbf{B}_{11}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})u_1(\mathbf{k})+\cdots+\mathbf{B}_{Nm}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N}\\&\quad+1)u_m(k-N+1)+\mathbf{C}_1^*\mathbf{U}(\mathbf{k})+\cdots+\mathbf{C}_N^*\mathbf{U}(\mathbf{k}-\mathbf{N}\\&\quad+1)+\mathbf{E}(\mathbf{k})\\&=(\mathbf{A}_1^*\mathbf{Q}_0+\mathbf{G}_2\mathbf{F}_0)+\mathbf{L}_{10}\mathbf{U}(\mathbf{k})+\mathbf{E}(\mathbf{k})\\&=\mathbf{Q}_1+\mathbf{L}_{10}\mathbf{U}(\mathbf{k})\end{aligned} \tag{4}$$

Continuing the above procedure, we obtain

$$\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+2)=\mathbf{Q}_2+\mathbf{M}_{10}^1\mathbf{V}_1(\mathbf{k},\mathbf{k}+1)+\mathbf{L}_{11}\mathbf{U}(\mathbf{k}+1)+\mathbf{L}_{20}\mathbf{U}(\mathbf{k}) \tag{5}$$

$$\begin{aligned}\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+3)&=\mathbf{Q}_3+\mathbf{N}_{10}^1\mathbf{V}_2(\mathbf{k},\mathbf{k}+2)+\mathbf{M}_{11}^1\mathbf{V}_1(\mathbf{k},\mathbf{k}+2)\\&\quad+\mathbf{M}_{20}^1\mathbf{V}_1(\mathbf{k},\mathbf{k}+2)+\mathbf{M}_{10}^2\mathbf{V}_1(\mathbf{k},\mathbf{k}+1)+\mathbf{L}_{12}\mathbf{U}(\mathbf{k}+2)\\&\quad+\mathbf{L}_{21}\mathbf{U}(\mathbf{k}+2)+\mathbf{L}_{30}\mathbf{U}(\mathbf{k})\\&\ \ \vdots\end{aligned} \tag{6}$$

where

$\mathbf{L}_{1j}=[\mathbf{B}_{11}^*\mathbf{Q}_j\ \mathbf{B}_{12}^*\mathbf{Q}_j \cdots \mathbf{B}_{1m}^*\mathbf{Q}_j]+\mathbf{C}_1^*$

$\mathbf{L}_{2j}=[\mathbf{B}_{21}^*\mathbf{Q}_j\ \mathbf{B}_{22}^*\mathbf{Q}_j \cdots \mathbf{B}_{2m}^*\mathbf{Q}_j]+\mathbf{C}_2^*+\mathbf{A}_1^*\mathbf{L}_{1j}$

$\mathbf{L}_{3j}=[\mathbf{B}_{31}^*\mathbf{Q}_j\ \mathbf{B}_{32}^*\mathbf{Q}_j \cdots \mathbf{B}_{3m}^*\mathbf{Q}_j]+\mathbf{C}_3^*+\mathbf{A}_1^*\mathbf{L}_{2j}+\mathbf{A}_2^*\mathbf{L}_{1j}$

$\mathbf{M}_{ji}^i=[\mathbf{B}_{i1}^*\ \mathbf{L}_{j1}\ \mathbf{B}_{i2}^*\ \mathbf{L}_{j1} \cdots \mathbf{B}_{im}^*\ \mathbf{L}_{j1}]$

$\mathbf{N}_{ji}^i=[\mathbf{B}_{i1}^*\ \mathbf{M}_{j1}^i\ \mathbf{B}_{i2}^*\ \mathbf{M}_{j1}^i \cdots \mathbf{B}_{im}^*\ \mathbf{M}_{j1}^i]$

$$\mathbf{Q}_j=\begin{cases}\sum_{i=0}^{j-1}\mathbf{A}_{j-i}^*\mathbf{Q}_i+\mathbf{G}_{j+1}\mathbf{F}_0+\mathbf{E}(\mathbf{k}); & j\le N-1\\ \sum_{i=1}^{N}\mathbf{A}_i^*\mathbf{Q}_{j-i}+\mathbf{E}(\mathbf{k}) & ;\ j\ge N\end{cases}$$

$$\mathbf{V}_1(\mathbf{k},\mathbf{k}+i)\begin{bmatrix}\mathbf{U}(\mathbf{k})u_1(k+i)\\ \vdots\\ \mathbf{U}(\mathbf{k})u_m(k+i)\end{bmatrix}$$

$$\mathbf{V}_1(\mathbf{k},\mathbf{k}+j)\begin{bmatrix}\mathbf{V}_{j-1}(\mathbf{k},\mathbf{k}+\mathbf{j}-1)u_1(k+j)\\ \vdots\\ \mathbf{V}_{j-1}(\mathbf{k},\mathbf{k}+\mathbf{j}-1)u_m(k+j)\end{bmatrix};\ j\ge 2$$

## 2. Design of the Controller

The computations involving the iterations of large dimension matrices cause numerical difficulties. For simplicity, we consider the case of one-step ahead prediction. The objective function is given by

$$\begin{aligned}J&=[\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)-\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1)]^T\Gamma[\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)\\&\quad-\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1)]+\mathbf{U}^T(\mathbf{k})\mathbf{B}\mathbf{U}(\mathbf{k})\end{aligned} \tag{7}$$

where

$\Gamma=\mathrm{diag}\{\gamma_1^2,\cdots,\gamma_n^2\}$

$\mathbf{B}=\mathrm{diag}\{\beta_1^2,\cdots,\beta_n^2\}$

We define

$\underline{\mathbf{Q}}=\mathbf{A}_1^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})+\cdots+\mathbf{A}_N^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N}+1)+\mathbf{E}(\mathbf{k})$

$\underline{\mathbf{H}}=\mathbf{B}_{21}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-1)u_1(k-1)+\cdots+\mathbf{B}_{2m}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-1)u_m(k-1)$
$\quad+\cdots+\mathbf{B}_{N1}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N}+1)u_1(k-N+1)+\cdots$
$\quad+\mathbf{B}_{Nm}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}-\mathbf{N}+1)u_m(k-N+1)$

$\underline{\mathbf{Z}}=\mathbf{C}_2^*\mathbf{U}(\mathbf{k}-1)+\cdots+\mathbf{C}_N^*\mathbf{U}(\mathbf{k}-\mathbf{N}+1)$

$\underline{\mathbf{R}}=[\mathbf{B}_{11}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})\cdots\mathbf{B}_{1m}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})]$

Then one-step ahead output $\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1)$ can be represented by

$$\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1)=\underline{\mathbf{Q}}+\underline{\mathbf{H}}+\underline{\mathbf{Z}}+(\mathbf{C}_1^*+\underline{\mathbf{R}})\mathbf{U}(\mathbf{k}) \tag{8}$$

Substitution of (8) into (7) gives

$$\begin{aligned}\mathbf{J}&=[\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)-\underline{\mathbf{Q}}-\underline{\mathbf{H}}-\underline{\mathbf{Z}}-(\mathbf{C}_1^*+\underline{\mathbf{R}})\mathbf{U}(\mathbf{k})]^T\Gamma\\&\quad[\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)-\underline{\mathbf{Q}}-\underline{\mathbf{H}}-\underline{\mathbf{Z}}-(\mathbf{C}_1^*+\underline{\mathbf{R}})\mathbf{U}(\mathbf{k})]+\mathbf{U}^T(\mathbf{k})\mathbf{B}\mathbf{U}(\mathbf{k})\end{aligned} \tag{9}$$

Minimization of (9) yields

$$\begin{aligned}\mathbf{U}(\mathbf{k})&=[(\underline{\mathbf{R}}\pm\mathbf{C}_1^*)^T\Gamma(\underline{\mathbf{R}}+\mathbf{C}_1^*)+\mathbf{B}]^{-1}(\underline{\mathbf{R}}+\mathbf{C}_1^*)^T\Gamma\{\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)\\&\quad-\underline{\mathbf{Q}}-\underline{\mathbf{H}}-\underline{\mathbf{Z}}\}\end{aligned} \tag{10}$$

Rearrangement of (10) gives

$$\begin{aligned}\mathbf{U}(\mathbf{k})&=\underline{\mathbf{W}}[\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1)-\mathbf{A}_1^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T})-\{\mathbf{Y}(\mathbf{k})-\mathbf{Y}^*(\mathbf{k})\}\\&\quad-\sum_{i=2}^{N}\{\mathbf{A}_i^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1-i)+\sum_{j=1}^{m}\mathbf{B}_{ij}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1-i)\\&\quad u_j(\mathbf{k}+1-i)+\mathbf{C}_i^*\mathbf{U}(\mathbf{k}+1-i)\}]\end{aligned} \tag{11}$$

where

$$\underline{\mathbf{W}}=[(\underline{\mathbf{R}}+\mathbf{C}_1^*)^T(\underline{\mathbf{R}}+\mathbf{C}_1^*)+\mathbf{B}]^{-1}(\underline{\mathbf{R}}+\mathbf{C}_1^*)^T\Gamma \tag{12}$$

## 3. Offset Compensator

At steady-state, (1) gives

$$\mathbf{Y}_s^*=\lim_{k\to\infty}\mathbf{Y}^*(\mathbf{k})=\mathbf{A}_s^*\mathbf{Y}_s+(\mathbf{B}_s^*+\mathbf{C}_s^*)\mathbf{U}_s \tag{13}$$

where

$$\mathbf{A}_s^* = \sum_{i=1}^{N} \mathbf{A}_i^*,\ \mathbf{B}_s^* = \sum_{i=1}^{N} \overline{\mathbf{B}}_i,\ \mathbf{C}_s^* = \sum_{i=1}^{N} \mathbf{C}_i^*$$
$$\overline{\mathbf{B}}_i = [\mathbf{B}_{i1}^*\mathbf{Y}_s \cdot \cdot \mathbf{B}_{im}^*\mathbf{Y}_s];\ 1 \le i \le N$$

and $\mathbf{Y}_s$ and $\mathbf{U}_s$ are steady-state values of output and input variables respectively. Sustitution of (13) into (11) yields upon rearrangement

$$\{(\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*) + \mathbf{B}\}\mathbf{U}_s = (\overline{\mathbf{B}} + \mathbf{C}_1^*)^T \Gamma (\mathbf{Y}_{ds} - \mathbf{Y}_s) + (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)\mathbf{U}_s \quad (14)$$

where $\overline{\mathbf{B}}_1 = [\mathbf{B}_{11}^*\mathbf{Y}_s \cdot \cdot \mathbf{B}_{1m}^*\mathbf{Y}_s]$. From (14), we can see that there is no offset if $\mathbf{B}=0$, and that nonzero $\beta_i$ always gives offset. As before, we introduce a constant offset compensation matrix $\mathbf{K} \in R^{n \times n}$ such that (11) becomes

$$\mathbf{U}(\mathbf{k}) = \underline{\mathbf{W}}[\mathbf{K}\mathbf{Y}_d(\mathbf{k}+\mathbf{T}+1) - (\mathbf{A}_1^* - \mathbf{A}_s^* + \mathbf{K}\mathbf{A}_s^*)\mathbf{Y}^*(\mathbf{k}+\mathbf{T}) - \mathbf{K}\{\mathbf{Y}(\mathbf{k}) - \mathbf{Y}^*(\mathbf{k})\} - \sum_{i=2}^{N} \{\mathbf{A}_i^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1-\mathrm{i}) + \sum_{j=1}^{m} \mathbf{B}_{ij}^*\mathbf{Y}^*(\mathbf{k}+\mathbf{T}+1-\mathrm{i})u_j(K+1-\mathrm{i}) + \mathbf{C}_i^*\mathbf{U}(\mathbf{k}+1-\mathrm{i})\}] \quad (15)$$

At steady-state, (15) becomes

$$\{(\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*) + \mathbf{B}\}\mathbf{U}_s = (\overline{\mathbf{B}}_1 + \mathbf{C}_1)^T \Gamma [\mathbf{K}(\mathbf{Y}_{ds} - \mathbf{Y}_s) + \{(\mathbf{K} - \mathbf{I})(\mathbf{B}_s^* + \mathbf{C}_s^*) + (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)\}\mathbf{U}_s] \quad (16)$$

Rearrangement of (16) gives

$$(\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\mathbf{Y}_{ds} - \mathbf{Y}_s) = \{\mathbf{B} - (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\mathbf{K} - 1)(\mathbf{B}_s^* + \mathbf{C}_s^*)\}\mathbf{U}_s \quad (17)$$

It is clear from (17) that zero offset is achieved if

$$\mathbf{B} - (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\mathbf{K} - 1)(\mathbf{B}_s^* + \mathbf{C}_s^*) = \mathbf{O}$$

or

$$\mathbf{B} + (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma (\mathbf{B}_s^* + \mathbf{C}_s^*) = (\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma \mathbf{K} (\mathbf{B}_s^* + \mathbf{C}_s^*) \quad (18)$$

If $n=m$, i.e., input and output vectors have the same dimensions, $\mathbf{K}$ has the explict form given by

$$\mathbf{K} = \mathbf{I} + \{(\overline{\mathbf{B}}_1 + \mathbf{C}_1^*)^T \Gamma\}^{-1} \mathbf{B} (\mathbf{B}_s^* + \mathbf{C}_s^*)^{-1} \quad (19)$$

## IDENTIFICATION ALGORITHM

Since an identification algorithm is itself an adaptation algorithm in the adaptive control system, the analysis of the identification problem with bounded disturbances has often been coupled with the analysis of adaptive control systems with bounded disturbances. Samson [1983] analyzed the identification methods for the discrete-time system subject to bounded disturbances. Identification for bilinear systems has been studied by Frick and Valavi [1978], Kubrusly [1981], Zhang [1983], Wang et al. [1987]. Yeo [1986] have used ARMA model in the identification of single variable bilinear systems.

A single variable bilinear system can be described by ARMA representation of a form

$$y(k) = p^T x(k-1) + d(k) \quad (20)$$

In order to identify the system parameter vector p, we propose a recursive identification algorithm of the form

$$p^*(k) = p^*(k-1) + \xi(k-1)x(k-1)e^*(k) \quad (21)$$

where

$$e(k) = y(k) - y^*(k|k)$$
$$e^*(k) = y(k) - y^*(k|k-1)$$
$$y^*(k|k) = p^{*T}(k)x(k-1)$$
$$y^*(k|k-1) = p^{*T}(k-1)x(k-1)$$

and the gain $\xi(k-1)$ is calculated as follows

$$\xi(k-1) = \begin{cases} \dfrac{2\lambda(k)[\zeta(k)-1]}{\zeta(k)\|x(k-1)\|^2 + \theta(k)} & ; \zeta(k) > 1 \\ 0 & ; \zeta(k) \le 1 \end{cases} \quad (22)$$

where

$$\zeta(k) = \frac{|e^*(k)|}{q^D}$$
$$0 < \lambda(k) \le 1$$
$$0 < \theta(k) < R_1 < \infty$$
$$1 \le q < R_2 < \infty$$

In this study the above identification algorithm is used and the extension to multivariable bilinear system is relatively straigthforward.

## EXAMPLES

To illustrate the proposed adaptive predictive control method for multivariable bilinear models, we present some simulation examples. To demonstrate the effect of tuning parameter $\beta$ and the usefulness of the offset compensator, the incorrect models is used in non-adaptive predictive control. To correct for the effect of model inaccuracy, we introduce a simple filter given by

$$\mathbf{U}(\mathbf{k}) = (1-\alpha)\mathbf{U}^*(\mathbf{k}) + \alpha\mathbf{U}(\mathbf{k}-1) \quad (23)$$

where $\mathbf{U}^*(\mathbf{k})$ is the unfiltered input vector from the control algorithm (15). In order to demonstrate the useful features of the identification algorithm used in this study, we present the result of identification in example 2. Results of simulations of the proposed adaptive predictive method are shown in example 1 and 3. In examples 1 and 3, the disturbance is assumed to be constant as $\mathbf{D}(\mathbf{k}) = [0.5\ 0.5]^T$. Constant matrices $\Gamma$ and $\mathbf{B}$ are used in the simulations, i.e., $\gamma_i = 1 (1 \le i \le n)$ and $\beta_i = \beta (1 \le i \le m)$.

### 1. Example 1

The process is given by (24) and the model of the process is assumed to be represented by (25).

$$\mathbf{Y}(\mathbf{k}) = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \mathbf{Y}(\mathbf{k}-1) + \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix} \mathbf{Y}(\mathbf{k}-2) + \begin{bmatrix} 0.5 & 0 \\ 0 & 1.2 \end{bmatrix} \mathbf{Y}(\mathbf{k}-1)u_1(k-4) + \begin{bmatrix} -0.12 & 0 \\ 0 & 0.3 \end{bmatrix} \mathbf{Y}(\mathbf{k}-2)u_1(k-5) + \begin{bmatrix} 4 & 1 \\ 2 & 1.5 \end{bmatrix} \mathbf{U}(\mathbf{k}-4) + \begin{bmatrix} 2 & 0.5 \\ -1.4 & 0.6 \end{bmatrix} \mathbf{U}(\mathbf{k}-5) + \mathbf{D}(\mathbf{k}) \quad (24)$$

$$\mathbf{Y}^*(\mathbf{k}) = \begin{bmatrix} -0.16 & 0 \\ 0 & 0.21 \end{bmatrix} \mathbf{Y}(\mathbf{k}-1) + \begin{bmatrix} 0.24 & 0 \\ 0 & 0.36 \end{bmatrix} \mathbf{Y}(\mathbf{k}-2) + \begin{bmatrix} 0.48 & 0 \\ 0.1 & 1.16 \end{bmatrix} \mathbf{Y}\mathbf{k}-1)u_1(k-4) \begin{bmatrix} -0.11 & 0.02 \\ 0.01 & 0.27 \end{bmatrix} \mathbf{Y}(\mathbf{k}-2)u_1(k-5) + \begin{bmatrix} 3.8 & 1.05 \\ 1.6 & 1.7 \end{bmatrix} \mathbf{U}(\mathbf{k}-4) + \begin{bmatrix} 2.2 & 0.43 \\ -1.31 & 0.52 \end{bmatrix} \mathbf{U}(\mathbf{k}-5) \quad (25)$$

The result of control for $\beta=0$ is shown in Fig. 1. For $\beta=0$, the controller is unstable. The results of control for $\beta=0.5$, 1.0, and 3.0 are shown in Fig. 2, 3, and 4. Increasing $\beta$ yields stable behavior, but the offset increases as $\beta$ increases. Fig. 5 show the effect of the offset compensator for $\beta=3.0$. The offset is eliminated by the introduction of the offset compensator, but the response is much more oscillatory and takes much longer to reach steady

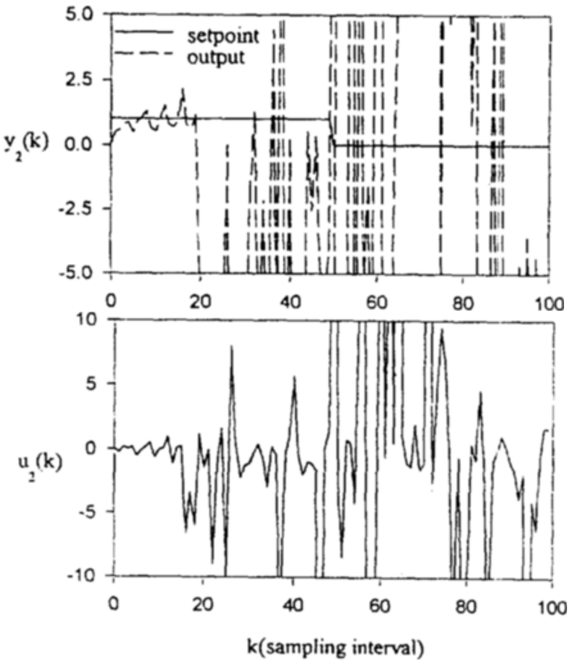


Fig. 1. Results of control for example 1 ($\beta=0.0$).

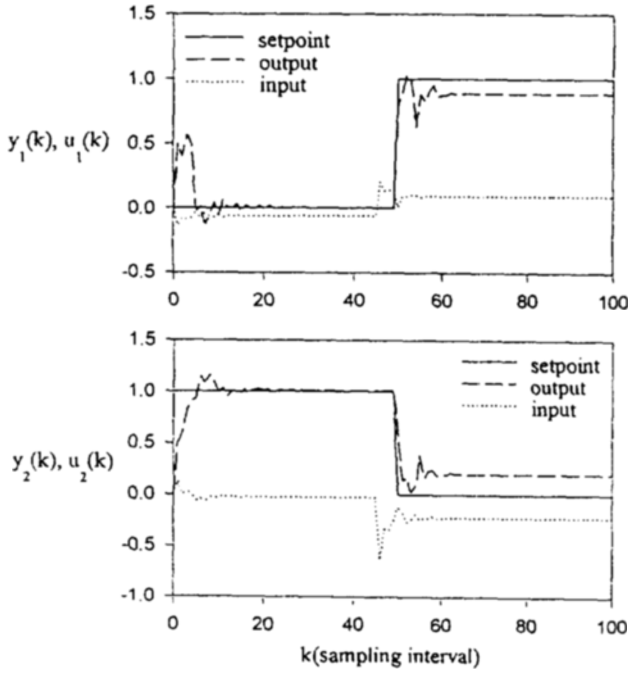


Fig. 2. Results of control for example 1 ($\beta=0.5$).

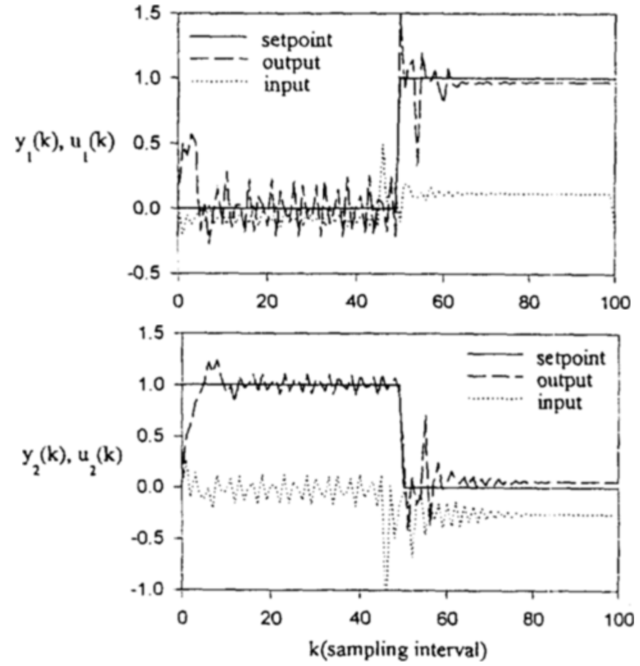


Fig. 3. Results of control for example 1 ($\beta=1.0$).

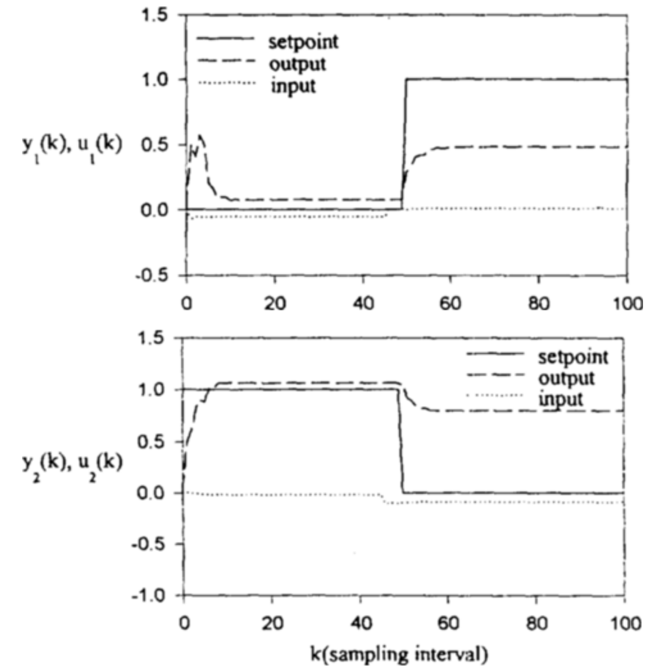


Fig. 4. Results of control for example 1 ($\beta=3.0$).

state. Fig. 6 shows the smoothing effect of the filter. As $\alpha$ is increasesd, the response becomes more damped and more sluggish, but it does not take significantly longer to reach steady state than it does when the compensator is used without the filter.

**2. Example 2**

The multivariable bilinear process used in this example is the same as that in the previous example. In the identification, the algorithm given by (20) and (21) with $q=1$, $\theta(k)=1$, and $\lambda(k)=\zeta(k)/2[\zeta(k)-1]$ was used. The inputs are pseudo-random binary sequences (PRBS) with an amplitude of 0.5. The ranges of disturbances are $-1.2\leq d_1(k)\leq 1.2$ and $-0.2\leq d_2(k)\leq 0.2$ respectively. The initial values of the model parameters correspond to non-zero process parameters and zero process parameters are set to 1.0 and 0.0 respectively. Fig. 7 shows the output tracking errors. We can see that the output tracking errors are confined within the expected bound of disturbances.

**3. Example 3**

The process used in this example is the same as that in the previous example. The process initially has zero inputs, outputs and disturbances. The initial values of the model parameters are

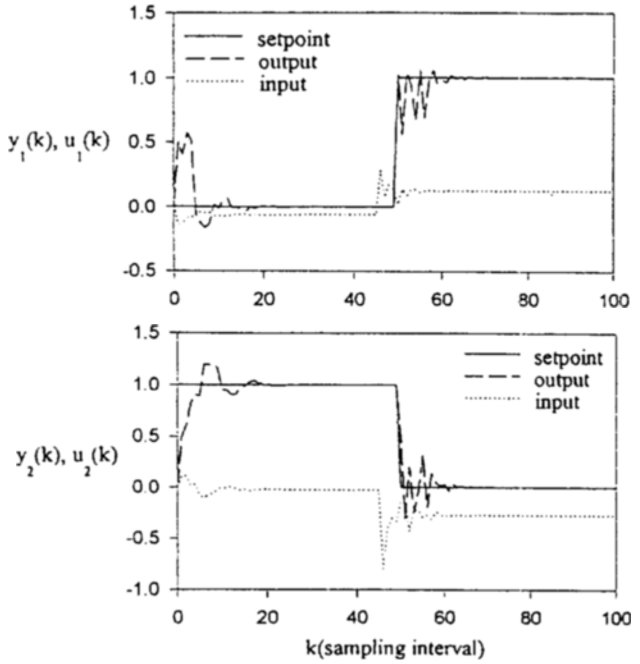


**Fig. 5. Effect of offset compensator for example 1 (β=3.0).**

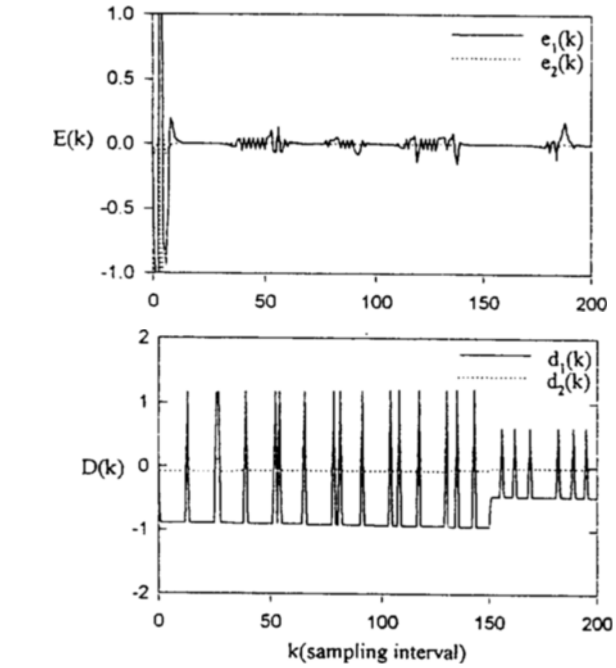


**Fig. 7. Tracking error of output and disturbances for example 2.**

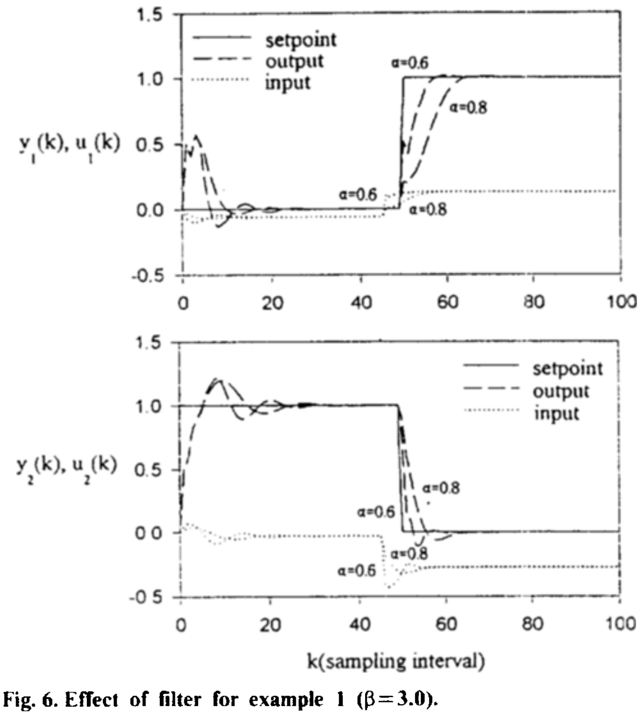


**Fig. 6. Effect of filter for example 1 (β=3.0).**

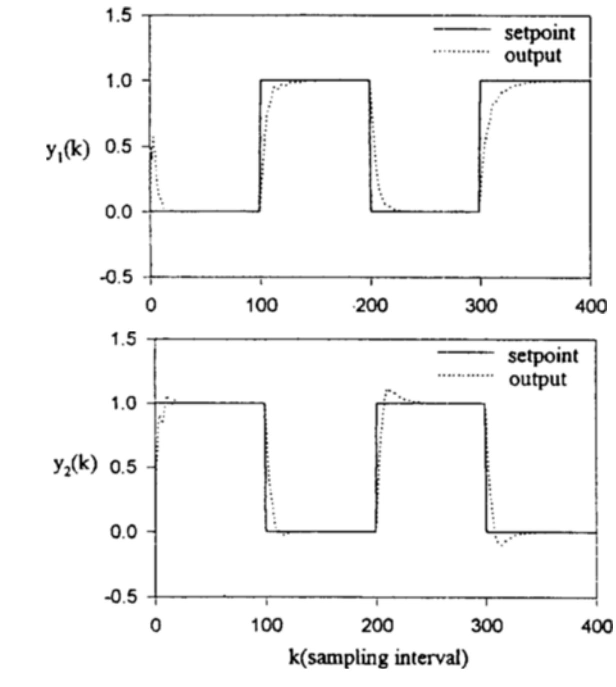


**Fig. 8. Results of control for example 3 (β=3.0, α=0.8).**

the same as that in example 2. The results of the adaptive predictive control are shown in Fig. 8. As we can see outputs track the set points very well.

## CONCLUSION

A predictive control methods for multivariable bilinear processes have been developed in this study. The controller uses a bilinear model, which makes possible a greater range of accurate representation of a general nonlinear process than is possible with a linear model. Representation of future outputs in terms of available data is complicated for multivariable bilinear processes, and one-step ahead prediction was employed in the present study. But, numerical simulation results show the satisfactory performance even with disturbances and incorrect model. The offset caused by the increase of tuning parameter β is eliminated by the offset compensator proposed in this study. A filter is used to reduce the oscillation caused by the introduction of the offset compensator. Simulation results show the effectiveness of the offset compensator and filter. Controller tuning is simple through the following adjustment of two parameters. The control algorithm parameter $\beta$ should be selected based on the assumption that the model is perfect. The filter parameter $\alpha$ should be selected to reduce the oscillation.

Any suitable recursive identification algorithms can be used in the present adaptive predictive control method. The equation error identification method extended to multivariable system was used in this study. By combining the previous identification algorithm with the predictive control method developed in this study, we could obtain the adaptive predictive control method for multivariable bilinear processes. Tuning and the theoretical study of robustness of adaptive predictive control method for multivariable bilinear process remain as major problems.

## ACKNOWLEDGEMENT

This research was supported in part by NON DIRECTED RESEARCH FUND, Korea Research Foundation.

## NOMENCLATURE

$\mathbf{A}_i, \mathbf{B}_{ij}, \mathbf{C}_i$ : process parameter matrices ($\in R^{n\times n}$, $\in R^{n\times n}$ and $\in R^{n\times m}$ respectively)
$\mathbf{A}_i^*, \mathbf{B}_{ij}^*, \mathbf{C}_i^*$ : model parameter matrices ($\in R^{n\times n}$, $\in R^{n\times n}$ and $\in R^{n\times m}$ respectively)
D : disturbance bound
**D** : disturbance vector, $\in R^{n\times 1}$
d : disturbance
**E** : output error vector, $\in R^{n\times 1}$
e, e*: control output error
**I** : unit matrix
**J** : objective function
**K** : offset compensation matrix, $\in R^{n\times n}$
**k** : time (sampling interval)
m : the number of input variables
**N** : process order
n : the number of output variables
p : process parameter vector
p* : model parameter vector
q : identification parameter
$R_i$ : constants
T : time delay
**U** : plant input vector, $\in R^{m\times 1}$
**U*** : unfiltered input vector, $\in R^{m\times 1}$
$\mathbf{U}_s$ : steady-state values of input vector, $\in R^{m\times 1}$
u : process input
x : process data vector
**Y** : plant output vector, $\in R^{n\times 1}$
**Y*** : model output vector, $\in R^{n\times 1}$
$\mathbf{Y}_d$ : output setpoint vector, $\in R^{n\times 1}$
$\mathbf{Y}_s$ : steady state values of output vector, $\in R^{n\times 1}$
y : process output
y* : model output

### Greek Letters

$\alpha$ : filtering parameter
**B**, $\Gamma$ : diagonal weighting matrices
$\beta_i$ : weight parameter on the input
$\gamma_i$ : weight parameter on the control error
$\zeta$ : normalized control output error
$\xi$ : gain identification algorithm
$\lambda$ : identification parameter
$\theta$ : identification parameter

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